Dav.d Sheik

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David Cumming

Rob.t Gord.s Hospital

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Vol. V R. G. Hosp.

February 3rd. 1813. Wednesday

No 4 B.

David Cumming

Wm. Cumming Hyde Shiff Jb.

Mr David Shee

Barter

Bartering is the exchanging of commodities; and as neither party is supposed to sustain any loss, when the commodities exchanged are not equal of value, the deficiency is made up by money &c.

Case I.

When the quantity of one commodity is known, with its value, or that of its integer; as also the value of the integer or rate of selling some other commodity to be given for it, to find the quantity of this; or having the quantity of the latter given, to find the rate of selling it. If the amount of the given quantity be unknown, calculate it in the shortest manner you can, for the given value of its integer; then find how much of the other quantity this amount will purchase, at the given rate of selling it; or if the quantity be given, from thence find the rate of selling it.

Barter.

Case II.

If the quantities of both commodities, with the rate of selling them, be given; to find what quantity of some other commodity or money must be given in case of an inequality of the amount of the first commodities. Calculate the amount of each of the two given commodities; then their difference is the money, or amount of the third commodity to be advanced; whose quantity, from thence and its rate, is easily found.

Case III.

When, in bartering, one commodity is rated above the ready-money price; to find the quantity and bartering price of the other commodity.

Say, As the ready-money price of the one is to its bartering price, so is that of the other to its barter price; then the quantity of the latter commodity may be found

Barter.

either from the ready-money or bartering prices

Examples.

1) How much tobacco at 1£ 16sh per cwt, must be given for 3 pipes of wine at 28£ 10 sh per pipe?

£ 1. 16 : £ 28. 10sh : : 3 pipes.

$$\begin{array}{r} 20 \\ \hline 36 \end{array} \qquad \begin{array}{r} 20 \\ \hline 570 \\ 3 \\ \hline \end{array}$$

$$\begin{array}{r} 36\,)\,1710\,(\,47 \text{ Cwt } 2 \text{ qr.} \\ 144 \\ \hline 270 \\ 252 \\ \hline 18 \\ 4 \\ \hline 36\,)\,72\,(\,2 \text{ qr} \\ 72 \\ \hline \end{array}$$

2) What weight of hops, at 1£ 12sh per cwt, must be given for 14 cwt 1 qr 18 lb of cheese, at 1£ 6sh per cwt?

£ 1. 6 :—

$$\begin{array}{rr|l} \text{qr} & & 14 \\ \hline 1 = & \frac{1}{4} & 18 \,.\, 4 \,.\, — \\ 14 = & \frac{1}{2} & — \,.\, 6 \,.\, 6 \\ 4 = & \frac{1}{7} & — \,.\, 3 \,.\, 3\frac{1}{7} \\ & & 11 \\ \hline & & 18 \,.\, 14 \,.\, 8\frac{1}{7} \end{array}$$

Carried over

Barter.

1. 12 : 1 : : 18 . 14 . 8 ½

20	20
32	314
12	12
387	4496
7	7
2688	) 31473

Cwt 1 . 2qr 238 $\frac{3}{8}$ ℔. Ans.

3) If 24 yds of cloth be given for 5 cwt 1qr of tobacco at £1. 18 per cwt, what is the cloth rated at per yard?

£1 . 18 .

		5
¼	9 . 10 .	
	— . 9 .	6
24	9 . 19 .	6
£	— . 8 .	3¾ D Answer.

4th) A and B would barter; A hath 40 yds of cloth at 7sh 4D per yd, B hath 28½ ℔ tea, at 11sh 6D per ℔: Whether must pay balance, and how much?

7 . 4	2) 11 . 6
10	28½
14 . 13 . 4	16 . 2 . 0
	5 . 9
	16 . 7 . 9
	14 . 13 . 4
	A must pay 1 . 14 . 5 D. Answer.

Barter.

5th) E and F barter: E gives to F 90 gallons of brandy, at 7 sh 8 D per gallon; for which F gives to E 10 guineas in money, and 500 lb of cotton: what is it valued at per lb?

lb sh D
500 : 7 . 8 : : 10 guineas.
90
34 . 10 . 0
10 . 10 . 0 £ sh D g
500) 24 . 0 . 0 (0 . 0 . 11 . 2 25 gr.
20
500) 480 (0 sh
12
500) 5760 (11 D
500
760
500
260
4
500) 1040 (2 g
1000
20) 40 (2
) 500 (25

90 gallons @ 7/8 £34 . 10 s
10 guis 10 - 10 s
500 lb @ 11 2/3 £2 . 18 . 4
£33 - 8 . 4

Dan Cummins

Loss and Gain.

Questions in this rule are such whose solutions determine the loss or gain upon commodities; of which questions there is a great variety; but they may be all easily solved from a little consideration, and the following proportion, viz:

That the gains or losses, are in proportion as the quantities of goods, &c.

Examples.

1st) Bought 5 cwt 3 qr 14 ℔ of chuse, at 1 £ 12 sh per cwt, and sold it again for 2 £ 0 - 8 D per cwt; what was the gain upon the whole?

		2.	0.	8
		1.	12.	0
		0.	8.	8
				5
2 =	2	2.	3.	4
1 =	2	—	4.	4
14 =	2	—	2.	2
		—	1.	1
	£	2.	10.	11 D. Answer.

2nd) If 5 c 3 qr 14 ℔ be bought for 9 £ 8 sh, and sold for 12 £ 18 sh 11 d; what is the rate of gain per cwt?

Loss and Gain

11 . 15 . 11
9 . 8 . 0
2 . 10 . 11

5 . 3½ : 2 . 10 . 11 :: 1

4 | 20 | 8
23 | 50 | 8
2 | 12
47 | 611
8
47 | 4888
12 | 104
5h 8 . 8 D. Answer

3rd) If 8 cwt 18 lb be bought for 45 £; at what rate per lb shall I sell it to gain 10 £ upon the whole?

8 . 0 . 18 : 45 :: 1 : 1 . 2 . 1 $\frac{351}{457}$

112 | 10 | 20
914 | 55 | 20
457

£ sh D

20
457) 1100 (0 . 1 . 2 . 1 $\frac{351}{457}$ q
550
457
93

12
457) 1116 (2
914
202

4
457) 808 (1 $\frac{351}{457}$
457
351
457

Loss and Gain.

4th) If 8 cwt 18 ℔ cost £45, at what rate per ℔ must it be sold, that the loss upon the whole may be 10 £?

8 . 0 . 18 : 45 : : 10

142
914
457

10
35
20
700
12

914) 8400
457) 4200 (9 D 87/457 q. Answer.
4113
87
457

5th) Bought hops at £4 . 16 sh per cwt; at what rate per cwt must I sell them, to gain 15 £ per cent?

100 : 15 : : 4 . 16

20
96
15
1452
115
5160
1452
1452
100) 16288 0
12) 1324 . 3 . 4
20) 110 . . 4
£ 5 . 10 . 4 . 4 3/5 q. Answer.

Loss and Gain.

6th) Bought hops at 4£ 16sh per cwt? at what rate per cwt must I sell them to lose 15£ per cwt?

100 : 85 :: 4. 16
20
96
85
480
768
100) 8160
20) 81. 7 3
£ 4. 1. 7 3 qr. Answer.

7th) If, when I sell cloth at 7sh per yd, I gain 10£ per cent.; what will be the gain per cent. when it is sold for 8sh 6D per yard?

7 : 8½ :: 100
8½
880
55
7) 0330
133. 11. 5 7
100. - -
33. 11. 5 7

Exchange.

By exchange is mēnt the bartering or exchanging of the money of one place for that of another, and, like the bartering of wares, it commonly consists in finding what quantity of the money of one place will be equal to a given sum of another, according to a given course of exchange.

By course of exchange is meant the variable sum of the money of one place which is proposed to be given for a constant piece or sum of that of another, to serve for the present, as a rate or proportion by which to exchange other sums; and it is sometimes above and sometimes below the par.

By the par of exchange is meant an intrinsic equality between two pieces or sums of money, one of which is the constant piece or sum to which the course is compared.

The money in the banks of foreign places is finer or purer than that which is current in them; and the difference between any sum, is valued in the one, and its value in the other, is called agio.

Exchange.

I. With Ireland, America, and the West-Indies.

Accounts are kept in Ireland, America, and the West-Indies, in pounds, shillings, and pence, as in England; and the exchange per cent. sterling; the par being 108£ 6sh 8D Irish per 100£ sterling, or 1£ 1sh 8D per pound: also 5£ sterling is accounted worth 7£ of the currency of the West-Indies, because of the grew plenty of foreign coins there.

Examples.

1st) London remits to Dublin 375£ 15sh: What must be received there, exchange at 110 per cent?

100 : 110 : : 375 . 15

```
          375 . 15
              20
          -------
          7515
            110
          -------
          75150
        7515
        -------
100 | 826650
 20 |   826.6.6
      £ 413..6 sh 6D.
```

Exchange.

2nd) Dublin remits London 413£6sh 6D: What must be received there, exchange at 110 per cent?

```
110 : 100 :: 413. 6. 6
                  20
               8266
                  12
              99198
                 100
110 | 9919800
12  |   90180
20  |    7515
     £ 375. 15 sh
```

3rd) London remits to Jamaica for 212£12sh 6D sterling: What must be received for it, exchange at 135 per cent?

```
100 : 135 : 212. 12. 6
20    27      20
             2252
                12
            51030
               27
           357210
          102060
20 | 1377810
12 |  68890 .. 2
20 |  5740. 10
     £ 287 .. .. 10½ D
```

Exchange.

4th) Jamaica remits to London for 287£ 10½D currency: What must be received for it exchange at 135 per cent?

```
137 :  100 :: 287 . . . 10½
 27     20      20
              5740
                12
            51030
                20
27 | 1377840
12 |  50030
20 |  14252 . 6
     £ 212. 12. 6 . 3 qr.
```

II. With Holland, Flanders, and Germany.

In these places accounts are sometimes kept in pounds, shillings, and pence, as in England, and sometimes in guilders, stivers, and pennings. The money of Holland and flanders is distinguished by the name flemish, and they exchange by the pound sterling, the par being 33 sh 4D flemish per pound sterling.

Exchange.

Note

16 pennings make one stiver
20 stivers or 40 pence - 1 guilder or floren
8 pennings - 1 grote or penny
12 grotes or pence - 1 shilling
20 shillings one pound

And in Germany.

12 pennings - 1 shilling lubs
16 lubsh shilling - 1 mark.
6 pennings one grote flemish
6 lubsh shil - 1 shill flem.
72 marks lubs - 1 pound flem.

Examples.

1st) To how much flemish will 700£ sterling amount, exchange at 34sh flem. per £ sterling?

20 : 34 : : 700

```
      700
20 ) 23800
```

£ 1190 flem.

2nd) To how much sterling will 1190£ flem amount exchange at 34sh per £ sterling?

34 : 20 : : 1190

```
      1190
34 ) 23800
```

£ 700 sterling

Exchange.

3rd) How much flemish must be given for 314£ 5sh sterling, exchange at 33sh 8D flem per £ ster.

```
  20 :   33.. 8 :: 314.. 5
  12      12          20
 ----    ----       ----
 240     404        6285
                     404
                   ------
                   25140
                 251400
              ---------
240 ) 2539140
  20 )  10579
      £ 528.. 19.. 9 D.
```

4th) How much sterling must be given for 528£ 19sh 9D flemish, exchange at 33sh 8D per pound ster?

```
  33.. 8 :  20 :: 528.. 19.. 9
  12        12          20
 ----      ----       ----
 404       240        10579
                         12
                     ------
                     126958
                        240
                     -------
                     5078320
                    253916
                  ---------
404 ) 30469920
  12 )    75420
  20 )     6285
      £ 314.. 5 sh
```

Exchange.

5th) How many guilders may I have for 173£ 14sh 2D sterling, exchange at 35 sh 3½ D per £ sterling?

20 :	35 . 3½ : :	173 . 11 . 2
12	12	20
240	423	3471
2	2	12
480	846	41690
		847

48 { 6) 35311430
8) 44139214
12) 1556517 5/6
20) 3678711 5/6
Guild. 18392 st. 11 5/6 pennings.

6th How much sterling must I have for 2714 guid 15 st, exchange at 35 sh 6D per £ sterling?

35½ :	20 : :	2714 . . 15
2	2	4
71	40	10859 . . 0
		110 . . 10
		108590
		40

71 | 4343600 61/71
12 | 611791 61/71
20 | 50983 . 1 61/71
£ 254 . 15 . 1 . 1 61/71

Exchange.

III. With France.

In France, accounts are kept in livers, sols, and deniers; exchange being made by the French crown, the par of which is 2sh 6D, or 2sh 6¼ more nearly?

Note. 12 deniers — a sol or sou, value 0.0 ½
20 sols — a livre - - - - 0.. 9¾
3 livres — 1 crown ~~or ecu~~ - - - 2. 5¼

Examples.

1st) How many livres, &c. will 121£ 18sh 6D amount to, exchange at 32⅞D per ecu?

32⅞ : 1 :: 121.. 18.. 6
8 8 20
263 8 2438
12
29262
24
117048
58524
263) 702288
Liv. 2670.. 5 sols.. 11 47/90 Den.

2nd) To how much sterling will 3956 livres amount, exchange at 31D per ecu?

Exchange.

$$\begin{array}{l} 1 : 31 :: 3956 \\ \underline{3} \qquad\qquad\quad \underline{31} \\ 3 \qquad\qquad\quad 3956 \\ \qquad\qquad\quad 11868 \\ \qquad\qquad 3\overline{)122736} \\ \qquad\qquad 12)\ 40912\tfrac{2}{3} \\ \qquad\qquad 20)\ 3406.6 \\ \qquad\qquad\quad £\ \underline{170..6..6\tfrac{2}{3}D} \end{array}$$

3rd) How much French crowns may I have for 102 £ 13sh $11\frac{1}{2}$D sterling, exchange at $31\frac{3}{8}$D p. crown?

$$\begin{array}{l} 31\tfrac{3}{8} :: 1 :: 102.13..11\tfrac{1}{2} \\ \underline{8} \qquad\quad \underline{8} \qquad\quad \underline{20} \\ 251 \qquad\quad 8 \qquad\quad 2053 \\ \qquad\qquad\qquad\qquad \underline{12} \\ \qquad\qquad\qquad\quad 24647 \\ \qquad\qquad\qquad\qquad\quad \underline{4} \\ \qquad\qquad 251)\ 197178\ (\ \underline{785..34\ sols.} \\ \qquad\qquad\quad \underline{1757} \\ \qquad\qquad\quad 2147 \\ \qquad\qquad\quad \underline{2008} \\ \qquad\qquad\quad 1398 \\ \qquad\qquad\quad \underline{1255} \\ \qquad\qquad\quad 143 \\ \qquad\qquad\qquad \underline{20} \\ \qquad\qquad\quad 2860 \\ \qquad\qquad\qquad\quad \underline{3} \\ \qquad\qquad 251)\ 8580 \\ \qquad\qquad\quad \text{sols } 34 \end{array}$$

Exchange

4th) To how much sterling will 1978 cr. 15 sols amount, exchange at 31 5/8 per crown?

$$1 : 31\tfrac{5}{8} :: 1978\tfrac{5}{12}$$

$$\begin{array}{r} 1 \\ 12 \\ \hline 12 \end{array} \qquad \begin{array}{r} 31\tfrac{5}{8} \\ 8 \\ \hline 253 \end{array} \qquad \begin{array}{r} 1978\tfrac{5}{12} \\ 12 \\ \hline 23741 \\ 253 \\ \hline 71223 \\ 118705 \\ 47482 \\ \hline 5008473 \end{array}$$

$$\begin{array}{r|l} 12 & 5008473 \\ 8 & 500839\tfrac{5}{12} \\ 12 & 62586 \\ 20 & 5214 \end{array}$$

£ 260. 14. 0 D

IV. With Spain

In Spain they keep their accounts in piastres, rials, and mervadies; reckoning 372 mervadies, to a rial, and 8 rials to a piastre by which they exchange, and its par is 4sh 6D sterling.

Note. In Genoa and Leghorn they keep their accounts in livres, sols, and deniers, as

Exchange.

in France, but exchange by the piastre, as in Spain, which in Genoa is accounted 5 livres, and at Leghorn 6. At Venice too accounts are by some kept in the same manner and by others in ducats and gross, reconing 24 gross to a ducat upon which they exchange, and its par is accounted 4s 4D sterling.

Examamples.

1st) How many piastres, &c. shall I receive in Spain for 510 £ sterling, exchange at 50D ster p. piastre?

$$50 : \begin{array}{r} 1 \\ 12 \\ \hline 12 \end{array} :: \begin{array}{r} 510 \\ 20 \\ \hline 10200 \\ 12 \\ \hline 50\,)\,122400 \\ \text{piastres } 2448 \end{array}$$

2nd) Spain draws upon London for 2448 pias. exchange at 50D per pias: How much ster will the draught amount to?

$$\begin{array}{r} 1 \\ 12 \\ \hline 12 \end{array} : 50 :: \begin{array}{r} 2448 \\ 50 \\ \hline 12\,)\,122440 \\ 20\,)\,10220 \\ £\,510 \end{array}$$

Exchange.

How many livres must be given at Genoa for 175 £15 sh ster exchange at 52 D ster. p. pia?

$$52 : 1 :: 175 . 15$$

$$\begin{array}{r} 5 \\ \hline 5 \end{array} \qquad \begin{array}{r} 20 \\ \hline 2515 \\ 12 \\ \hline 42180 \\ 5 \\ \hline 52\overline{)210900} \end{array}$$

Livres $\underline{4055 . 15 . 4\frac{3}{13}}$

4th Genoa draws upon London for 3000 livres, How much ster. will satisfy this draught exchange at $50\frac{1}{2}$ D per piastre?

$$1 : 50\tfrac{1}{2} :: 3000$$

$$\begin{array}{r} 5 \\ \hline 5 \end{array} \qquad \begin{array}{r} 50\tfrac{1}{2} \\ \hline 150000 \\ 1500 \\ \hline 5\overline{)151500} \\ 12\overline{)30300} \\ 20\overline{)2525} \end{array}$$

£ $\underline{126 . 5 \text{ sh}}$

Exchange.

V. With Portugal.

In Portugal accounts are kept in milreas, & reas, reckoning 1000 reas to a milrea, as its name imports; and they exchange by the milrea, the par of which is about 6s 8D ½, or 6s 9D sterling.

Examples.

1st) To how many milreas will 715£ amount, exchange at 5s 8D per milrea?

$$\begin{array}{r} 5.8: \\ 12 \\ \hline 68 \end{array} \qquad \begin{array}{r} 715: \\ 20 \\ \hline 14300 \\ 12 \\ \hline 68\overline{)171600} \end{array} \qquad \begin{array}{r} 1 \\ 12 \\ \hline 12 \end{array}$$

Milr. $\underline{2527.529\frac{7}{17}}$ reas.

2nd) To how many £, &c. will $\overset{\text{milreas}}{2523}.\overset{\text{reas}}{528}$ amount exchange at 5s 8D per milrea?

$$\begin{array}{r} 1: \\ 12 \\ \hline 12 \end{array} \qquad \begin{array}{r} 5.8 \\ 12 \\ \hline 68 \end{array} :: \begin{array}{r} 2523.528 \\ 1000 \\ \hline 2523528 \\ 68 \\ \hline 1000\overline{)17199904} \\ 12\overline{)17199.3\frac{111}{125}} \\ 20\overline{)1429.9.11} \end{array}$$

£ $\underline{714.19.11.3\frac{111}{125}}$

Exchange.

3rd) How many milreas must be given for 213£ 7.10D, exchange at 5s 9½D per milrea

$$\begin{array}{rcl} 69\tfrac{1}{2} : & 1 : : & 213.\ 7.10 \\ \underline{2} & \underline{2} & \underline{20} \\ 139 & 2 & 4267 \\ & & \underline{12} \\ & & 51214 \\ & & \underline{2} \\ & 139) & 102428 \end{array}$$

milr. 736. 892 $\frac{12}{139}$ reas.

4th How much sterling will 736 milreas amount, exchange at 5Sh 9D½ per milrea?

$$\begin{array}{rcl} 1 : & 69\tfrac{1}{2} : & 730 \\ \underline{2} & \underline{2} & \underline{139} \\ 2 & 139 & 6624 \\ & & 2208 \\ & & \underline{730} \\ & 2 & 102304 \\ & 12 & 51152 \\ & 20 & 4262.8 \end{array}$$

£ 213. 2. 8 D

Arbitration of Exchange.

As the course of exchange between one nation & another, is almost continually varying, either by rising or falling, from the variations in the circumstances and balance of trade; so the design of arbitration is to remit or draw upon foreign places in such a manner, as shall turn out the most profitable.

Arbitration is generally divided into two parts, Simple & Compound.

I. Simple arbitration.

1. In simple arbitration, the exchanges among three places only are conserned. The par of arbitration, or arbitrated price, is such a rate of exchange between two places as shall be in proportion with the rates assigned between each of them, and a third place. After this par of arbitration is commuted, by comparing it with the present course of exchange, a person can judge which way to remit or draw to the most advantage, and deter-

Simple Arbitration.

mine what the advantage shall be.

Examples.

1st) If the exchange between London & Amsterdam be 33s 9D per £ ster, and the exchange between London & Paris be 32D per crown; required the par of arbitration between Amsterdam and Paris.

As	20	:	$33\frac{3}{4}$	::	32
	80	:	135	::	32
	16	:	27	::	32
	1	:	27	::	2
			2		
			54 Flemish per Crown.		

2nd) If the exchange betwan Amsterdam & Paris be 54D per crown, & between Amsterdam & London 33s 9D per £ sterling; required the arbitrated price between Paris & London.

As	$33\frac{3}{4}$	:	20	::	54
	135	:	80	::	54
	27	:	16	::	54
	1	:	16	::	2
			2		
			32D per Crown.		

Simple

3rd/ If the exchange from London to Paris be 32D per crown, & to amsterdam 405D per £ sterling and if, by advice from Holland or France, the course of exchange between amsterdam & paris be fallen to 52D per crown; what may be gained per cent. by London drawing on the one place & remitting to the other.

As 1 : 405 :: 100 : 40500 @ Amsterdam
52 : 1 :: 40500 : $\frac{40500}{52}$ @ Paris.
1 : 32 :: $\frac{40500}{52}$: $\frac{8 \times 40500}{13}$ = $\frac{32400}{13}$ =
£ 103 . 16 . $11\frac{5}{13}$
100
to ans: 3 . 16 . $11\frac{5}{13}$ By drawing at Paris and remitting may be gained.

4th/ London is indebted to Petersburgh 5000 rubles, Now the exchange between Petersburgh & England is at 50D per ruble, between Petersburgh & Holland @ 90D per ruble, and between Holland

Arbitration.

and England at 3£ 6s 4D, which will be more advantageous for London to be drawn upon.

As 1 : 50 : : 5000 : 250000 from Pet. to Lond.
1 : 90 : : 5000 : 450000 from Pet. to Amster.
$36\frac{1}{3}$: 20 : : 450000 : 247706 $\frac{46}{109}$ from Ams. to London.
109 : 60

60

109)2700000(247706 $\frac{46}{109}$
520 250000
840 12 2293 $\frac{63}{109}$
770 20 191..1
700 9..11..1 $\frac{63}{109}$
46

II. Compound Arbitration

Compound Arbitration respects the cases in which the exchanges between three, four, or more places are concerned.

A person who knows at what rate he can draw or remit directly, and also hath advis of the course of exchange in foreign places, may trace out a path for circulating his money, through more or fewer of such places, and also in such order, as to make a benefit of his skill & credit: And herein lies the great art of such negociations.

But to determine in what order, & through how many places to circulate a bill, no general rule can be given, as

Compound. Arbitration.

it depends intirely upon a persons judgment, and a close attention to the results of former cases of the like kind.

The directions necessary for determining whether a direct or an assigned circular draft shall be preferable, or contained in the following

Rules.

1. Distinguish the given rates or prices in the circular course, into antecedents & consequents; and place the antecedents in one column, & the consequents on another, on the right, fronting one another, by way of equation. And in this distribution into antecedents & consequents, each consequent must be of the same kind with ye next antecedent, and ye first antecedent of

Compound

the same kind with the last consequent, which must be the sum whose value in exchange is required.

2. Multiply the antecedents continually for a divisor, and the consequents continually for a dividend; & the quotient of the products will be the value of the sum required by such exchange.

3. Then compute its value by the direct exchange, or by any other circular exchange, & by comparing the values together, you will perceive the most advantageous method.

Examples.

1. If London would remit 1000£ sterling to Spain, the direct exchange being 42½d per piastre of 272 mervadies, it is required whether will be more, profitable, the direct remittance or by remitting first to Holland, at 35s per £; thence to

Arbitration.

France, @ 58D per Crown; thence to Venice, @ 100 crowns per 60 ducats; & thence to Spain, at 360 mervadies per ducat.

Antecedents.	Consequents.
1£ sterling	= 35s or 420D flemish.
58D flemish	= 1 crown
100 crowns	= 60 ducats
1 ducat	= 360 mervadies.
272 mervadies	= 1 piastre

How many piastres = 1000£ sterling

Then $\frac{420 \times 60 \times 360 \times 1000}{58 \times 100 \times 272} = \frac{210 \times 30 \times 45 \times 10}{29 \times 17}$

$= \frac{2835000}{493} = 5750\frac{250}{493}$ piastres = the value of 1000£ by the circular exchange.

But 422D : 1 piastre :: 1000£, or 24000D :

$\frac{480000}{85} = \frac{96000}{17} = 5647\frac{1}{17}$ piastres, the value by the direct exchange.

So that the circular exchange is the more advantageous, & it produces $103\frac{13}{29}$ pias. more than the other.

Involution.

A power is a number produced by multiplying any given number continually by itself a certain number of times.

Any number is called the first power of itself; if it be multiplied by itself, the product is called the second power, & sometimes the square; if this be multiplied by the first power again, the product is called the third power; and sometimes the cube; & if this be multiplied by the first power again, the product is called the fourth power & so on; that is, the power is denominated from ye number which exceeds the multiplications by 1.

Thus: 3 is the first power of 3.

$3 \times 3 = 9$ is the second power of 3.

$3 \times 3 \times 3 = 27$ is the third power of 3.

$3 \times 3 \times 3 \times 3 = 81$ is the fourth power of 3.

&c. &c.

Involution.

Involution is the finding of powers, to do which, from their defination there evedently comes this

Rule.

Multiply the given number, or first power, continually by itself, till the number of multiplications be 1 less than the index of the power to be found, & the last product will be the power required.

Examples.

1. What is the second power of 45?

```
  45
  45
 225
180
2025
```

2. What is the square of 416?

```
   416
   416
  2496
 416
1664
173056
```

Involution.

3. What is the square of ·027?

$$
\begin{array}{r}
\cdot027 \\
\cdot027 \\
\hline
189 \\
54 \\
\hline
\cdot000729
\end{array}
$$

4. What is the third power 3·5?

$$
\begin{array}{r}
3\cdot5 \\
3\cdot5 \\
\hline
175 \\
105 \\
\hline
12\cdot25 \\
3\cdot5 \\
\hline
6125 \\
3675 \\
\hline
42875
\end{array}
$$

5. What is the second power of $\frac{2}{3}$?

$$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$$

6. What is the third power of $\frac{5}{9}$?

$$\frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} = \frac{125}{729}$$

Involution.

7. What is the fourth power of 71·8?

```
      71·8
      71·8
     5744
     718
    5026
   5155·24
   5155·24
  2062096
 1031048
2577620
2577620
515524
2577620
26576499·4576
```

8. What is the square of $3\frac{2}{5}$?

$$3\frac{2}{5} = \frac{3 \times 5 + 2}{5} = \frac{17}{5} =$$

$$\frac{17 \times}{5 \times} \frac{17}{5} = \frac{289}{25}$$

Involution.

9. What is the fifth power of ·029?

```
          ·029
          ·029
          ----
           261
           58
          ----
           841
          ·029
          ----
          7569
         1682
         -----
         24389
          ·029
         ------
         219501
         48778
         ------
         707281
           ·029
         -------
         9365529
        1414562
        -------
·0000000205111149
```

Evolution.

The root of any given number, or power is such a number as being multiplied by itself a certain number of times, will produce the power; and it is denominated the first, second, third, fourth, &c. root, respectively, as the number of multiplications made of it to produce the given number is 0, 1, 2, 3, &c.; that is, the name of the root is taken from the number which exceeds the multiplications by 1, like the name of the power in Involution.

Evolution is the finding of the roots of numbers, either accurately, or in decimals to any proposed extent. The power is first to be prepared for extraction, or evolution, by dividing it from the place of units, to the left-hand in integers, and to the right in decimal fractions, into periods, containing each as many places of figures as are denonominated by the index of the root. if the power contain a complete number of such periods, if it do not, the defect will be either on the

Evolution.

right-hand or left, or both; if the defect be on the right-hand, it may be supplied with annexing ciphers, and after this whole periods of ciphers may be annexed to continue the extraction with, if necessary, but if there be a defect in the left, such defective periods must remain unaltered, & it is accounted the first period of the given number, just the same as if it were complete. Now this division may be conveniently made by making a point over the place of units, and also over the last figure of every period on both sides of it; that is, over every second figure if it be the second root, over every third if it be the third root, &c.

Thus, to point this number 21035896·12735;
for the second root, it will be 21035896·127350;
but for the third root, thus 21035896·127350;
and for the fourth, thus 21035896·12735000.

Evolution.

To extract the Square root.

1. Having pointed the given number into periods of two figures each, find, from the table of powers in page , or otherwise a square number either equal to, or the next less than the first period; which subtract from it, and set the root of the square on the right hand side of the given number, after the manner of quotient in Division, for the first figure of the root required.

2. To the remainder annex the second period for a dividend; and on the left-hand of it write the doube of the root already found, after the manner of a division.

3. Consider what figure, which if annexed to the divisor, and the result multiplied by it. the product may be equal to, or the next less than the dividend, and it will be the next figure of the root.

Evolution.

4. From the dividend subtract the product, & to the remainder bring down the next period, for a new dividend: to which, as before, find a divisor by doubling the figures already found in the root; and from these find the next figure of the root as in the last article; and continue the operation still in the same manner, till all the periods be used, or as far as you please.

Examples.

1. What is the root of 2025?

```
        2025(45 Root.
        16
   85) 425
       425
```

2. What is the root of ·000729?

```
        ·000729(027 Root.
          4
   47)   329
         329
```

Evolution.

3. What is the root of 2?

```
                2 (1414213 56237 Root
                1
        24) 100
            96
       281)  400
             281
      2824)  11900
             11296
     28282)   60400
              56564
    282841 )  383600
              282841
   2828423)  10075900
              8485269
  2828426)  1590631 (56237
             176418
               6712
               1055
                206
                  8
```

4. What is the root of 17·3086?

Evolution.

17·3056(4·16 Root.

16

81) 130

81

826) 4956

4956

5. What is the root of 3?

3,000000000000(1,732050 Root.

1

27) 200

189

343) 1100

1029

3462) 7100

6924

34641(5) 176000

173225

346410(0) 2775000

6. What is the root of 5?

5(2.236068 root.

4

42) 100

84

1600 Carried over

Evolution.

Brought forward 443)1600
1329
4466) 27100
26796
4466) 304(068
26

7. What is the root of 11?

11(3·3166625 root.
9
63) 200
189
661) 1100
661
6626) 43900
39756
6632) 4144(625
105
32

Rules for the square roots of Vulgar Fractions & Mixt Numbers.

First prepare all vulgar fractions by reducing them to their least terms, both for this and all other roots. Then:

Evolution.

1. Take the root of the numerator, and the denominator for the respective terms of the root required. And this is the best way if the denominator be a complete power, but if it be not, then

2. Multiply the numerator & denominator together; take the root of the product: this root being made the numerator to the denominator of the given fraction, or made the denominator to the numerator of it, will form the fractional root required.

That is, $\sqrt{\frac{a}{b}} = \frac{\sqrt{ab}}{b} = \frac{a}{\sqrt{ab}}$

And this rule will serve whether the root be finite or infinite.

3. Or reduce the vulgar fraction to a decimal, and extract its root.

4. Mixt numbers may be reduced to improper fractions, and extracted by the first or second rule;

Evolution.

or the vulgar fraction may be reduced to a decimal, then joined to the integer, and the root of the whole extracted.

Examples.

1. What is the root of $\frac{25}{36}$?

$$\sqrt{\frac{25}{36}} \text{ is } \frac{5}{6} \text{ Root.}$$

2. What is the root of $\frac{27}{147}$?

$$\sqrt{\frac{27}{147}} = \frac{3}{7} \text{ Root.}$$

3. What is the root of $\frac{9}{12}$?

$$\frac{9}{12} = \frac{3}{4} = 75(0866\,025 \text{ Root.}$$

$$\begin{array}{r} 64 \\ 166)\,1100 \\ 946 \\ 1726)\,10400 \\ 10356 \\ 17320)\,44\,(025 \\ 9 \end{array}$$

4. What is the root of $17\frac{3}{8}$?

Old Evolution. New

$17\frac{3}{8} = 17.3750(4.168333$ Root.

```
        16
  81) 137
        81
 826) 5650
      4956
8328) 69400
      66624
83363) 2776(333
```

By means of the square root, also, we readily find the 4th root, or the 8th root; or the 16th root, &c.; that is, the root of any power whose index is some power of the number 2; viz., by extracting so often the square root as is denoted by that power of 2; that is, two extractions for the fourth, three for the 8th root, and so on.

```
21035.8000(145.037237(12.0431407 root:
   1                1
24) 110          22) 45
    96                44
285) 1435        2414) 10372
     1425              9616
29003) 108000      24083) 75637
    6   87009          6) 72249
        20991(7237         3388(1407
          687                980
          107                 17
           20
```

Deer's Evolution. Elgin

To Extract the Cube Root.

1. By the common rule.

par le rule commune

1. Having divided the given number of three figures each, find the nearest less cube to the first period, in the third line of the table of powers in page ; set its root in the quotient, and subtract the same cube from the figure period; to the remainder bring down the second period, and call this the resolvend.

2. To three times the square of the root, just found, add three times the root itself, setting this one place more to the right than the former, and call this sum the divisor. Then divide the resolvend wanting the last figure, by the divisor, for the next figure of the root, which annex to the former; calling this last figure e, and the part of the root before found called a.

3. Add all together these three products, namely, thrice a square multiplied by e, thice a multiplied by e square, and e cube, setting each of them one place more to the right than the former; and call the sum the subtrahend, which

must not exceed the resolvend; and if it does, then make the last figure 0 less, and repeat the operation for finding the subtrahend.

4. From the resolvend take the subtrahend, & to the remainder join the next period of the given number for a new resolvend; to which form a new divisor from the whole root now found; and from thence another figure of the root, as directed in article 2, &c.

Example.

To extract the cube root of 48228·544.

$3 \times 3^2 = 27$	48228·544	(36·4 root.
$3 \times 3 = 09$	27	
Divisor 279	21228	resolvend.
$3 \times 3^2 \times 6 = 162$		
$3 \times 3^2 \times 6^2 = 324$		add
$6^3 = 216$		
$3 \times 36^2 = 3888$	10656	subtrahend
$3 \times 36 = 108$		
98988	1572544	resolvend
$3 \times 36^2 \times 4 = 15552$		
$3 \times 36 \times 4^2 = 1728$		add
$4^3 = 64$		
	1572544	subtrahend
	0000000	remainder.

Evolution.

II. To extract the cube root by a short way.

1. By trials take the nearest rational to the given cube or number, whether it be greater or less, and call it the assumed cube.

2. Then the sum of the given number and double the assumed cube, will be to the sum of the assumed cube and double the given number as the root of the assumed cube is to the root required, nearly. Or as the first sum is to the difference of the given or the assumed, so is the assumed root to the difference of the roots nearly.

3. Again, by using in like manner, the cube of the root last found, as a new assumed cube, another root will be got still nearer. And so on as far as you please; using always the cube of the last found root, for the assumed cube.

Examples

To find the Cube Root of 210348.

Evolution.

must not exceed the resolvend; and if it does, then make the last figure e less, and repeat the operation for finding the subtrahend.

4. From the resolvend take the subtrahend, & to the remainder join the next period of the given number for a new resolvend; to which form a new divisor from the whole root now found; and from thence another figure of the root, as directed in article 2, &c.

Example.

To extract the cube root of 48228·544.

$3 \times 3^2 = 27$ | 48228·544 (36·4 root.
$3 \times 3 = 09$ | 27
Divisor 279 | 21228 resolvend.

$3 \times 3^2 \times 6 = 162$
$3 \times 3^2 \times 6^2 = 324$ } add
$6^3 = 216$

$3 \times 36^2 = 3888$ | 19656 subtrahend
$3 \times 36 = 108$
98988 | 1572544 resolvend

$3 \times 36^2 \times 4 = 15552$
$3 \times 36 \times 4^2 = 1728$ } add
$4^3 = 64$

1572544 subtrahend
0000000 remainder.

Evolution.

Brought forward.
Cts 63106·43729 : diff. ·481355 :: 27·6047 :
the diff. ·000210834

Conseq. the root req, is 27·6049108 34

To extract any root whatever.

Let G be the given power or number, n the index of the power, A the assumed power, r its root, R the required root of G.

Then as the sum of n + 1 times A and n − 1 times G, is to the sum of n +, 1 times G and n − 1 times A, so is the assumed root r, to the required root R.

Or, as half the said sum of n + 1 times A and n − 1 times G, is to the difference between the given and assumed powers, so is the assumed root r to the difference between the true and assumed roots: which difference, added to or subtracted gives the true root nearly.

Evolution.

That is, $\overline{n+1}.\ A + \overline{n-1}.\ G + \overline{n+1}.\ G + \overline{n-1}.\ A :: r : R$

Or, $\overline{n+1}.\ \frac{1}{1} A + \overline{n-1}.\ \frac{1}{2} G : A \backsim G :: r : R \backsim r$;

And the operation may be repeated as often as you please, by using always the last found root for the assumed root, and its n th power for the assumed power A.

Ex. To extract the 5th root of 21035·8.

Here it appears that the 5 root is between 7·3 & 7·4. Taking 7·3, its 5th power is 20730·71593.

Hence, then, we have

$G = 21035{\cdot}8;\ r = 7{\cdot}3;\ n = 5;\ \frac{1}{2}\overline{n+1} = 3;\ \frac{1}{2}\overline{n-1} = 2.$

$A = 20730{\cdot}716$

$G - A = 305{\cdot}084$

$A = 20730{\cdot}716 \quad G = 21035{\cdot}8$

$\qquad\qquad 3 \qquad\qquad\qquad 2$

$3A = 62192{\cdot}148 \qquad 42071{\cdot}6$

$2G = 42071{\cdot}6$

As $104263{\cdot}7 : 305{\cdot}084 :: 7{\cdot}3 : {\cdot}0213605$

$$\begin{array}{r} 7{\cdot}3 \\ \hline 915252 \\ 2135588 \\ \hline 104263{\cdot}7\,)\,2227{\cdot}1132\,(\,{\cdot}0213605 \text{ the diff.} \\ 14184 \\ 3758 \\ 630 \\ 5 \end{array}$$

Evolution.

measure

General Rules for extracting any root out of a Vulgar Fraction or mixt number.

1. If the given fraction have a finite root of the kind required it is best to extract the root out of the numerator and denominator for the terms of the root required.

2. But if the fraction be not a complete power it may be thrown into a decimal and then extracted. Or

3. Take either of the terms of the given fraction for the corresponding term of root: For the other terms of the root extract the required root of the product, and using from the multiplication of such a power of the first assigned term of the root whose index is less by 1 than that of the given power by the other term of the given number. This rule will do when the root is either finite or infinite. That is, for any root in general.

Evolution.

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \frac{\sqrt[n]{ab^{n-1}}}{b} = \frac{a}{\sqrt[n]{ba^{n-1}}}$$

4. Mixt numbers may be reduced either to improper fractions or decicimals, and then extracted.

Examples on the cube &c.

1. What is the 3rd root of 2?

It is soon found the root of 2 lies between 1·2 & 1·2.
Taking there fore 1·2 its cube is 1·728.
Hence 1·728

```
      2
  3·456   2·
  2·      1·728
  5·456 : ·272 :: 1·2 : 1·2
          1·2          ·0598
  5456 ) ·3264 ( ·00598
          536
           45
```

Again, assume the root = 1·26. Its cube is 2000376.
Hence 2000376

```
          2
  4000752   2·000376
  2         2
  6·000752 : ·000376 :: 1·26 : 1·26
                1·26          ·00007895
               2256           1·25992105
               752
              376
  6·000752 ) ·00047376 ( ·00007895
                 5371
                  570
                   30
```

Uimmmm 851

Evolution.

2. What is the 4th root of 2?

2000000, &c. (1·41, 42; 14, (1·189 207 Ans.

1	1
24)100	21)41
96	21
281) 400	228) 2042
281	1824
2824)11900	2369)21812
11296	21321
2828) 604	2378)493
38	417
10	

3. What is the 4th root of 97·41?

97·41, 00, 00 (9·86, 96, 50, 5 (3·14159999 Answer

81	9
188)1641	61)86
1504	61
1966)13700	624)2596
11296	2496
19729) 190400	6281)10050
177561	6281
19738)12839	62825)376950
996	314125
9	62830) 62825
	6278
	623
	58

Evolution.

4. What is the 5th root of 2?

Assume it = 1, whose 5th power is also 1.

$G = 2;\ r = 1;\ n = 5;\ \frac{1}{2}\overline{n+1} = 3$

$A = 1 \qquad \frac{1}{2}\overline{n-1} = 2$

$G - A = 1$

$A = 1$

3

$3A = 3$

$2G = 4$

As $7 : 1 :: 1 : \frac{1}{7} = \cdot 143$ nearly $= R - 1$.

And, $R = 1\frac{1}{7}$ nearly.

Again, Assume $1\frac{1}{7} = \frac{8}{7}$. Its 5 power is $\frac{32768}{16807}$ Hence

$G = 2 = \frac{33614}{16807} \qquad 3 = A = \frac{98304}{16807}$

$A = \frac{32768}{16807} \qquad 2G = \frac{67228}{16807}$

$G - A = \frac{846}{16807} \qquad 3G - 2A = \frac{31076}{16807}$

$165532 : 846 :: \frac{8}{7} : \frac{8 \times 846}{7 \times 165532} = \frac{1692}{28680} = 0\cdot 005842$

To which add $1\frac{1}{7} = 1\cdot 142857$

Gives $1\cdot 148699$ Ans.

Evolution

5. What is the 6th root of 21035·8?

The root is soon found to be nearly 5 whose 6th power is 15625.

Here $G = 21035·8$; $r = 5$; $n = 6$; $n \times 1 = 7$
$A = 15625$ $\qquad n - 1 = 5$

$7G \times 5A = 225375·6$

$7A \times 5G = 21455·4 : 225375·6 :: 5 : 5·252$ or $5\frac{1}{4}$ nearly.

Again, Assume $r = 5\frac{1}{4} = \frac{21}{4}$ Then its 6th power is

$A = \frac{85766121}{4196}$; but $G = 21035·8 \frac{86162 36·8}{4096}$ And

Then omitting the denominators.

$7A = 600362847$; $7G = 6038457·6$
$5G = 430813184$; $5A = 428830605$

$1031176031 : 1031969062·6 :: 5\frac{1}{4} : 5·250037$ Ans.

Evolution.

6: What is the 7th. root of 210358?

Here $n=7$; $n+1=8$, and $n-1=6$; and their halves are 4 and 3. The root is nearly 4, whose 7 power is $A=16384$; but $G=210358$ Hence

$4A=65536$; $4G=841432$

$3G=\underline{631074}$; $3A=\underline{49152}$

As 1286434 : 1332952 :: $4 : \frac{83}{20}$ or $4\frac{3}{20}$ root nearly.

Again, Assume $r=\frac{83}{20}$ whose 7 powers $A=$

$\frac{27136050989627}{1280000000}$; but $G=210358=\frac{269258240000000}{1280000000}$

Then, omitting the denominator,

$4A=1085442204$, &c. $4G=1077032960$ &c.

$3G=\underline{807774720}$ &c. $3A=\underline{81408153}$ &c.

As 1893216760 &c. : 1891114490 &c. :: $\frac{83}{20}$: 4145392 Ans.

Evolution.

7. What is the cube root of $2\frac{8}{27}$?

The cube root of $\frac{8}{27}$ is $\frac{2}{3}$. Answer.

8. What is the 4th root of $\frac{80}{405}$?

The 4th root of $\frac{80}{405}$ or of $\frac{16}{81}$ is $\frac{2}{3}$

9. What is the cube root of 2?

Here the root is nearly 8, whose cube is A = 512
Also G = 5. Hence

2A 1024; 2G = 10
G = 5 A = 512

As 1524 : 1512 :: 8 : 7937007 The root very near at one operation, erring only in the last figure, which should be 5 instead of 7.

10. What is the cube root of $2\frac{10}{27}$?

The cube root of $2\frac{10}{27}$ of of $\frac{64}{27}$ is $\frac{4}{3}$.

Evolution

1. What is the 3 root of 75?

Here the root is nearly 2, its cube being A = 8.

Also G = 7. Hence

2A = 1.2; 2G = 144

G = 7.2; A = 8

As 2.3.2 : 2.2.4 :: 2 : $\frac{56}{29}$ or $\frac{58}{30}$ nearly.

Next take r = $\frac{58}{30}$ its cube is A = $\frac{195112}{27000}$.

Also G = 7.2 = $\frac{194000}{27000}$. Hence, omitting the denominator

2A = 390224; 2G = 388800

G = 194400; A = 195112

As 584624 : 583912 :: $\frac{58}{30}$: 1.930979 Answer.

No more at present but remains

No more at